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GSQ/D-20 **1053**

MATHEMATICS
BM-351
Real Analysis

Time : Three Hours] [Maximum Marks : 40

Note : Attempt *Five* questions in all. Select *one* question from each Section. Q. No. **1** is compulsory.

Compulsory Question

1. (a) If $f(x) = x, x \in [0, 1]$ and $P = \left\{0, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, 1\right\}$ be the partition of [0, 1], then compute L(*f*, P) and U(*f*, P). **2**

(b) Prove the inequality $1 \le \int_0^1 e^{x^2} dx \le e$. **1½**

(c) Show that $\int_1^\infty \frac{\sin x}{x^m}$ converges absolutely if $m \ge 1$. **1½**

(d) Show that the space (0, 1] with usual metric space is not complete. **1½**

(e) Prove that usual metric space (R, *d*) is not compact. **1½**

Section I

2. (a) Show that the function f defined by $f(x) = x,\ x \in [0, 1]$ is integrable and $\int_0^1 f(x)\,dx = \frac{1}{2}$. **4**

(b) By definition, prove that $\int_0^a \cos x\,dx = \sin a$, where a is a fixed number. **4**

3. (a) If a function f is continuous on $[a, b]$ and $F(x) = \int_a^x f(t)\,dt$, then F is differentiable on $[a, b]$ and $F' = f$. **4**

(b) Evaluate $\int_0^1 \sqrt{1 + x^4}\,dx$ by using mean value theorem. **4**

Section II

4. (a) Examine the convergence of the improper integral :

$$\int_{-a}^{a} \frac{x\,dx}{\sqrt{a^2 - x^2}}$$ **4**

(b) Show that the integral $\int_0^\infty x^{n-1} e^{-x} dx$ is convergent if $n > 0$. **4**

5. (a) Evauate $\int_0^a \frac{\log(1+\alpha x)}{1+x^2}\,dx, \alpha > 0$. **4**

(b) Show that $\int_0^{\pi/2} \sin x \log(\sin x)\,dx$ is convergent with the value $\log\left(\frac{2}{e}\right)$. **4**

Section III

6. (a) The interior set of a subset of a metric space is the largest open set contained in A. Prove. **4**

(b) If A and B are subsets of a metric space (X, *d*), then prove that : **4**

(i) $(A \cap B)^o = A^o \cap B^o$

(ii) $A^o \cup B^o \subset (A \cup B)^o$

7. (a) Every Cauchy sequence is bounded in a metric space. Prove. **4**

(b) Let X be a metric space, then prove that :

(i) any intersection of closed sets in X is closed.

(ii) finite union of closed sets in X is closed. **4**

Section IV

8. (a) A metric space is sequentially compact iff every infinite subset has a limit point. **4**

(b) Prove that continuous image of a compact metric space is compact. **4**

9. (a) A continuous image of a connected space is connected. Prove. **4**

(b) Every compact (sequentially-compact) metric space is complete. Prove. **4**