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GSQ/D-20 **1055**

MATHEMATICS

BM-353

Numerical Analysis

Time : Three Hours] [Maximum Marks : 30

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory.

(Compulsory Question)

1. (a) Show that $\Delta^2_{y,z} x^2$ is independent of *x, y, z.* **1½**

(b) State Gauss's Forward Interpolation Formula. **1½**

(c) Define Poisson's distribution. **1½**

(d) State Simpson's 1/3rd quadrative formula. **1½**

Unit I

2. (a) State and prove Newton-Gregory Formula. **3**

(b) Find the value of an annuity at $5\frac{3}{8}\%$, given the following table : **3**

Rate per cent	4	$4\frac{1}{2}$	5	$5\frac{1}{2}$	6
Annuity value	17.29203	16.28889	15.37245	14.53375	13.76483

3. (a) Given the following data, find $f(x)$ in polynomial process of $(x-5)$: **3**

x	0	2	3	4	7	9
$f(x)$	4	26	58	112	466	922

(b) The values of function $f(x)$ for values of x are given as $f(1)=4,\ f(2)=5,\ f(7)=5,\ f(8)=4$.

Find values of $f(6)$ and also value for x for which $f(x)$ is maximum or minimum. **3**

Unit II

4. (a) Derive Gauss Backward Interpolation formula. **3**

(b) Apply Bessel's formula for finding the values of y for $x = 3.75$ given that : **3**

$f(2.5) = 24.145,\ f(3.0) = 22.043,\ f(3.5) = 20.225,$
$f(4.0) = 18.644,\ f(4.5) = 17.262,\ f(5.0) = 16.047.$

5. (a) Find the probability distribution of the number of doublets in 4 throws of a pair of dice. **3**

(b) A manufacturer of bulb knows that 5% of his production is defective. If he sells bulbs in boxes of 100 and guarantees that not more than 4 bulbs will be defective, what is the approximate probability that a box will fail to meet the guaranteed quality ? (Take $e^{-5} = 0.0067$) **3**

Unit III

6. (a) The distance(s) covered by a car in a given time (t) is given in the following data : **3**

Time (minutes)	12	14	18	20	24
Distance (km)	14	18	23	25	34

Find the acceleration of the car at $t = 17$.

(b) Find the largest eigen values and the corresponding eigen-vector of the matrices : **3**

$$\begin{bmatrix} -1 & 1 & 2 \\ 0 & 1 & -1 \\ 4 & -2 & -9 \end{bmatrix}$$

7. Transform the matrix $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ to tri-diagonal form by Given's method. Find the eigen vector corresponding to the largest eigen value from the eigen vectors of the tri-diagonal matrix. **6**

Unit IV

8. (a) Evaluate $\int_0^4 e^x dx$, by Simpson's one-third rule using the data : **3**

$e = 2.72$, $e^2 = 7.39$, $e^3 = 20.09$, $e^4 = 54.60$

(b) Apply Runge-Kutta method to solve $\frac{dy}{dx} = x + y$; $y(0) = 1$ for $x = 0.1$. **3**

9. (a) Solve the following by Euler's modified method $\frac{dy}{dx} = \log_{10}(x + y)$, at $x = 1.2$ and 1.4 with $h = 0.2$, given $y(0) = 2$. **3**

(b) Use Picard's method to find the third approximation of the following differential equation : **3**

$$\frac{dy}{dx} = y - 1,\ y(0) = 2$$