Roll No. .... Total Pages : 4

GSM/D-20 **913**

ADVANCED CALCULUS

Paper - BM-231

Time allowed : 3 Hours *Maximum Marks : 40*

Note : Attempt five questions in all, selecting one question from each unit. Question No. 1 is compulsory. All questions carry equal marks.

Compulsory Question

1. (i) Write the statement of Lagrange's mean value theorem. 2

(ii) State Schwarz theorem. 2

(iii) Define screw-curvature. What is its magnitude. 2

(iv) Define osculating plane. 2

UNIT-I

2. (i) Every function defined and continuous on a closed interval attains its bounds in that interval. Prove it. 4

(ii) Verify Lagrange's mean value theorem for $f(x) = \sin x$ in $\left[\frac{\pi}{2}, \frac{5\pi}{2}\right]$. 4

3. (i) Show that : 4

$$\lim_{x \to 0} \frac{(1+x)^{\frac{1}{x}} - e + \frac{ex}{2} - \frac{11}{24}ex^2}{x^3} = -\frac{7e}{16}$$

(ii) Show that the function defined by $f(x) = x^2$ is uniformly continuous in $[-2, 2]$. 4

UNIT-II

4. (i) Show that the function f defined by :

$$f(x, y) = \begin{cases} \dfrac{x^3 - y^3}{x^2 + y^2} & ; \quad (x, y) \neq (0, 0) \\ 0 & \quad (x, y) = (0, 0) \end{cases}$$

is continuous at (0, 0). 4

(ii) State and prove Euler's theorem. 4

5. (i) Let $f : \mathrm{R}^2 \to \mathrm{R}$ be defined as :

$$f(x, y) = \begin{cases} \dfrac{xy}{x^2 + y^2} & ; \quad (x, y) \neq (0, 0) \\ 0 & \quad (x, y) = (0, 0) \end{cases}$$

Show that $\lim\limits_{(x, y) \to (0, 0)} f(x, y)$ does not exist. 4

(ii) If $z = 2u^2 - v^2 + 3w^2$, where

$u = xe^y$, $v = ye^{-x}$, $w = \dfrac{y}{x}$

Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$. 4

UNIT-III

6. (i) Show that the function

$$f(x, y) = \begin{cases} \dfrac{x^3 - y^3}{x^2 + y^2} & ; \quad (x, y) \neq (0, 0) \\ 0 & \quad\;\; (x, y) = (0, 0) \end{cases}$$

is continuous and possesses first order partial derivatives but not differentiable at the origin. 4

(ii) A rectangular box, open at the top, is to have a volume of 27/2 cubic ft. Find the dimensions of the box requiring least material for construction. 4

7. (i) Find the volume of the largest rectangular parallelopiped that can be inscribed in the ellipsoid 4

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

(ii) Give an example of a function f(x, y) for which $fxy^{(0, 0)} \neq fyx^{(0, 0)}$. 4

UNIT-IV

8. (i) Find the normal form of the curve

$2 \cos t\, \hat{i} + 2 \sin t\, \hat{j} + 6\, t\, \hat{k}, -\infty\ t < \infty.$ 4

(ii) Prove that : $\dfrac{d\hat{n}}{ds} = i\hat{b} - k\hat{t}.$ 4

9. (i) Show that the radius of spherical curvature of a circular helix $x = a \cos\theta$, $y = a \sin\theta$, $z = a\,\theta \cot\alpha$ is equal to the radius of circular curvature. 4

(ii) Find the involutes and evolutes of circular helix

$x = a \cos u$; $y = a \sin u$, $z = a\,u \tan\alpha$. 4