

Roll No. ....

Total Pages : 04

BCA/D-20

1192

MATHEMATICAL FOUNDATION-I

BCA-113

Time : Three Hours]

[Maximum Marks : 80

**Note :** Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. **1** is compulsory.

**(Compulsory Question)**

1. (a) Verify that  $(A \cap B)' = A' \cup B'$ , where  $A = \{2, 3, 4, 5, 6\}$ ,  $B = \{3, 6, 7, 8\}$  are subsets of  $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$ . **4**
- (b) Find  $r$ , if  ${}^{10}P_{r+1} : {}^{11}P_r = 30 : 11$ . **3**
- (c) Find  $\frac{dy}{dx}$ , when  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ . **3**
- (d) Show that  $x^2 + 4y = 0$  is a solution of **3**

$$\left(\frac{dy}{dx}\right)^2 + x\frac{dy}{dx} - y = 0$$

- (e) Solve the differential equation : **3**

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + y = 0.$$

### Unit I

2. (a) In a class of 1000 students, 625 students pass in Mathematics and 525 pass in English. How many students pass in Mathematics only and how many pass in English only ? **8**
- (b) In a set of integers, let a relation R be defined as  $aRb$  if and only if  $a-b$  is even. Prove that R is an equivalence Relation. **8**
3. (a) Find the number of arrangements that can be made out of the letter of the word PERMUTATION. In how many of these 5 vowels are together ? **8**
- (b) A polygon has 44 diagonals. Find the number of its sides. **8**

### Unit II

4. (a) Using  $\epsilon$ - $\delta$  definition, prove that : **8**
- $$\lim_{x \rightarrow a} \cos x = \cos a, \text{ where } a \in \mathbb{R}.$$
- (b) Differentiate :
- $$\tan^{-1} \frac{\sqrt{1+x^2}-1}{x} \text{ w.r.t. } \sin^{-1} \frac{2x}{1+x^2}. \quad \mathbf{8}$$

5. (a) If  $x^p y^q = (x+y)^{p+q}$ , prove that : **8**

$$\frac{dy}{dx} = \frac{y}{x}$$

- (b) If  $y = e^{\tan^{-1} x}$ , prove that : **8**

$$(1+x^2)y_2 + (2x-1)y_1 = 0.$$

### Unit III

6. (a) Find the differential equation of the family of the curves  $y = Ae^{3x} + Be^{5x}$ , where A and B are arbitrary constants. **8**

- (b) Solve the differential equation : **8**

$$(1+x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$$

7. (a) Solve the differential equation : **8**

$$(y \log x - 1) y dx = x dy$$

- (b) Verify that the differential equation :

$$x dy + y dy = a^2 \frac{(x dy - y dx)}{x^2 + y^2}$$

- is exact and solve it. **8**

#### Unit IV

8. (a) Solve the differential equation : 8

$$\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + \frac{dy}{dx} + y = \sin 2x$$

- (b) Solve the differential equation : 8

$$\frac{d^2y}{dx^2} + y = x - e^{2x}$$

9. (a) Solve the differential equation : 8

$$x^2 \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} - 4y = x^4$$

- (b) Solve the differential equation : 8

$$(3x+2)^2 \frac{d^2y}{dx^2} + 3(3x+2) \frac{dy}{dx} - 36y = 3x^2 + 4x + 1$$