Roll No. .... Total Pages : 4

GSE/D-20 **782**

ALGEBRA

Paper–BM-111

Time : Three Hours] [Maximum Marks : 40

Note : Attempt *five* questions in all, selecting *one* question from each section. Question No. 1 is compulsory.

Compulsory Question

1. (a) The diagonal elements of a Hermitian matrix are all real. 1½

(b) Show that no skew-symmetric matrix can be of the rank 1. 1½

(c) If α is an eigen value of a non-singular matrix A, then prove that $\frac{|A|}{\alpha}$ is an eigen value of adj. A. 1½

(d) If α, β, γ are the roots of the equation $2x^3 + x^2 + x + 1 = 0$, then find the value of $\sum \frac{1}{\alpha^2}$. 2

(e) Apply Descarte's rule of signs to discuss the nature of the roots of the equation $x^6 - 3x^2 - x + 1 = 0$. 1½

SECTION–I

2. (a) Every square matrix A can be expressed in one and only one way as P + ιQ, where P and Q are Hermitian matrices. 4

(b) Using elementary transformations, find the inverse of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$ and verify your answer.

4

3. (a) Prove that the characteristic roots of a Skew-Hermitian matrix are either zero or purely imaginary. 4

(b) Obtain the minimal equation of the matrix $A = \begin{bmatrix} 2 & 3 & 4 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix}$ and show that it is non-derogatory.

4

SECTION–II

4. (a) Find the value of k such that the following system of equations has a non-trivial solution :

$$x + ky + 3z = 0$$
$$4x + 3y + kz = 0$$
$$2x + y + 2z = 0.$$
4

(b) If A is an orthogonal matrix and if B = AP, where P is non-singular then prove that PB^{-1} is orthogonal. 4

5. (a) Reduce the quadratic form

$x_1^2 - 2x_2^2 + 3x_3^2 - 4x_2x_3 + 6x_3x_1$ to canonical form and find the rank, index and signature of the form. Also, find the equations of linear transformations. 4

(b) Reduce X'AX to the form $\sum_{i=1}^{n} k_i y_i^2$, where

$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -2 & -4 \\ 3 & -4 & -3 \end{bmatrix}$. Also, find the equations of transformation. 4

SECTION–III

6. (a) Find the condition that the sum of two roots of the equation $x^4 + px^3 + qx^2 + rx + s = 0$ is equal to zero. 4

(b) Show that the same transformation can remove both second and fourth terms of the equation

$x^4 + 16x^3 + 83x^2 + 152x + 84 = 0$

and hence solve it. 4

7. (a) Solve the equation $x^4 - 9x^2 + 4x + 12 = 0$, given that it has a multiple root. 4

(b) If α, β, γ are the roots of the equation

$x^3 + px^2 + qx + r = 0$,

find the equation whose roots are

$\alpha^2 - \beta\gamma, \beta^2 - \gamma\alpha, \gamma^2 - \alpha\beta$. 4

SECTION–IV

8. (a) Solve the equation $x^3 + x^2 - 16x + 20 = 0$ by Cardan's method. 4

(b) Apply Descarte's method to solve the equation

$$x^4 - 10x^3 + 35x^2 - 50x + 24 = 0.$$ 4

9. (a) Show that the roots of equation $x^3 - 3x + 1 = 0$ are

$$2\cos\frac{2\pi}{9},\ 2\cos\frac{8\pi}{9},\ 2\cos\frac{14\pi}{9}.$$ 4

(b) Solve the equation $x^4 - 4x^3 - 4x^2 - 24x + 15 = 0$ by Ferrari's method. 4