Roll No. ..... Total Pages : 3

GSE/D-20 **783**

MATHEMATICS

(Calculus)

Paper–BM-112

Time : Three Hours] [Maximum Marks : 40

Note : Attempt *five* questions in all, selecting *one* question from each section. Q. No. 1 is compulsory.

Compulsory Question

1. (a) Evaluate $\lim_{x \to \frac{5}{2}} [x]$. 2

(b) Define curvature of curve at a point. 1

(c) Evaluate $\int_0^{\pi/2} \sin^6 x \cos^4 x dx$. 1

(d) Find length of loop of the curve $r = a(\theta^2 - 1)$. 2

(e) If $y = e^{m \cos^{-1} x}$ prove that $(1 - x^2)y_2 - xy_1 - m^2 y = 0$. 2

SECTION–I

2. (a) Show that function defined by $f(x) = |x| + |x - 1|$ is continuous at $x = 0$ and 1. 4

(b) If $y = \left[\log\left(x + \sqrt{1 + x^2}\right)\right]^2$ prove that

$$(1 + x^2) y_{n+2} + (2n + 1) x y_{n+1} + n^2 y_n = 0 .$$ 4

3. (a) Apply Maclaurin's theorem to prove that

$$\sec x = 1 + \frac{x^2}{\lfloor 2} + \frac{5x^4}{\lfloor 4} + \cdots$$ 4

(b) If $f(x) = x^3 + 8x^2 + 15x - 24$, calculate the value of

$f\left(\frac{11}{10}\right)$ by Taylor's theorem. 4

SECTION–II

4. (a) Find all asymptotes of the curve.

$$x^3 + 3x^2y - 4y^3 - x + y + 3 = 0.$$ 4

(b) Find the asymptotes of curve $r \cos \theta = a \cos 2\theta$. 4

5. (a) Find radius of curvature for the curve $x = a \cos^3 \theta$, $y = a \sin^3 \theta$. 4

(b) Find points of inflexion for the curve $x = (\log y)^3$. 4

SECTION–III

6. (a) If $u_n = \int_0^{\frac{\pi}{2}} x^n \sin x \, dx \; (n > 1)$ show that

$$u_n + n(n-1)u_{n-2} = n\left(\frac{\pi}{2}\right)^{n-1}$$ and evaluate u_5. 4

(b) Trace the curve $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$. 4

7. (a) Find the intrinsic equation of cardioid $r = a(1 - \cos\theta)$. 4

(b) Find the length of the arc of parabola $y^2 = 4ax$ from the vertex to an extrimity of latus rectum. 4

SECTION–IV

8. (a) Find the area of curve $x = a\cos^3 t$, $y = b\sin^3 t$. 4

(b) Find the area common to the circle $r = a$ and the cardioid $r = a(1 + \cos\theta)$. 4

9. (a) Find the volume of the solid formed by the revaluation about x-axis of the curve $y^2(a + x) = x^2(3a - x)$. 4

(b) Show that the surface area of solid of revalution of $r = a(1 + \cos\theta)$ about initial line is $\frac{32}{5}\pi a^2$. 4