Roll No. .... **Total Pages : 04**

GSQ/D-20 1030

MATHEMATICS

Reals Analysis

BM-351

Time : Three Hours] [Maximum Marks : 27

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. **1** is compulsory.

1. (a) Compute L(*f*, P) and U(*f*, P) for the function $f(x) = \frac{1}{x^2}$ on [1, 4] and partition P = {1, 2, 3, 4}. **1½**

(b) Examine the convergence of $\int_1^\infty \frac{dx}{x}$. **1**

(c) Define open sphere and closed sphere and give examples. **1½**

(d) Show that in a discrete metric space (X, *d*), every subset of X is open. **1½**

(e) Show that in a metric space (X, *d*), the complement of every singleton set is open. **1½**

Section I

2. (a) Prove that a bounded function having a finite number of points of discontinuity on $[a, b]$ is integrable on $[a, b]$. **2½**

(b) Show that $\lim_{n\to\infty}\left[\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+.....+\frac{1}{2n}\right]=\frac{\pi}{4}$.

2½

3. (a) If f is bounded and integrable on $[a, b]$, then $|f|$ is also integrable on $[a, b]$. More over

$\left|\int_a^b f dx\right| \le \int_a^b |f|\, dx$. **2½**

(b) Evaluate the integral :

$$\int_{-1}^{1}\left([x]-x\right)dx$$

where $[x]$ stands for greatest integer not greater than x. **2½**

Section II

4. (a) Show that $\int_0^\infty \left(\frac{1}{1+x}-e^{-x}\right)\frac{dx}{x}$ is convergent. **2½**

(b) Examine the convergence of the integral $\int_0^\infty \frac{\cos x}{\sqrt{x^2+x}} dx$. **2½**

5. (a) Find the values of m and n for which the integral $\int_0^1 x^n e^{-mx} dx$ converges. **2½**

(b) Prove that :

$$\int_0^{\pi/2} \frac{dx}{\left(a^2 \sin^2 x + b^2 \cos^2 x\right)^2} = \frac{\pi\left(a^2+b^2\right)}{4a^3b^3}.$$ **2½**

Section III

6. (a) Prove that any metric space, (X, d), bounded or not, can be converted into a bounded metric space (X, d^*), where $d^*(x, y) = \frac{d(x,y)}{1+d(x,y)}$. **2½**

(b) Prove that every open sphere in a metric space (X, d) is an open set. **2½**

7. (a) Let (Y, d^*) be a subspace of a metric space (X, d). A subset B of Y is d^*-open iff there exists a d-open subset G of X such that B = G $\cap$ Y. **2½**

(b) Prove that the usual metric space (R, d) is complete. **2½**

Section IV

8. (a) Prove that every contraction mapping $f : (X, d) \to (X, d)$ is uniformly continuous on X. **2½**

(b) Prove that a compact subset of a metric space is closed and bounded. **2½**

9. (a) Prove that every closed subset of a compact metric space is compact. **2½**

(b) If E is connected subset of a metric space (X, d) such that ECA∪B, where A and B are separated sets in X, then either E⊂A or E⊂B. **2½**