Roll No. .... Total Pages : 4

GSE/D-20 **784**

SOLID GEOMETRY

Paper–BM-113

Time : Three Hours] [Maximum Marks : 40

Note : Attempt *five* questions in all, selecting *one* question from each unit. Question No. 1 is compulsory.

Compulsory Question

1. (a) What curve is represented by the equation $x^2 + 2xy + y^2 - 1 = 0$? 1½

(b) Find the equation of plane which cuts the sphere $x^2 + y^2 + z^2 = 25$ in a circle, whose centre is (2, 1, 2). 2

(c) Find the equation to the cone with vertex at origin and which passes through the curves $x^2 + y^2 = 4$, $z = 2$. 1

(d) Define Director circle. 1½

(e) Find the equations of the tangent planes to the surface $x^2 - 2y^2 + 3z^2 = 2$ which are parallel to the plane $x - 2y + 3z = 0$. 2

UNIT–I

2. (a) Find the lengths and equations of axes of the conic $5x^2 - 24xy - 5y^2 + 14x + 8y - 16 = 0$. 4

(b) Find the pole of the line $5x + 5y + 8 = 0$ w.r.t. the conic $2x^2 + 8xy + 3y^2 - 2x + 6y + 1 = 0$. 4

3. (a) Prove that the conics $x^2 + 3y^2 - 1 = 0$ and $2x^2 + 12xy + 39y^2 - 2x - 12y = 0$ have double contact with each other. Find the co-ordinates of the points of intersection of the tangents at points of contact. 4

(b) Prove that the two conics $\frac{l_1}{r} = 1 + e_1 \cos\theta$ and $\frac{l_2}{r} = 1 + e_2 \cos(\theta - \alpha)$ will touch each other if

$$l_1^2 (1 - e_2^2) + l_2^2 (1 - e_1^2) = 2l_1 l_2 (1 - e_1 e_2 \cos \alpha).$$ 4

UNIT–II

4. (a) A plane through a fixed point (1, 1, 1) cuts the axes in A, B, C. Prove that the locus of the centre of the sphere OABC, where O is the origin is given by $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 2$. 4

(b) Find the equation of the sphere, which touches the sphere $x^2 + y^2 + z^3 - x + 3y + 2z - 3 = 0$ at the point (1, 1, –1) and passes through the origin. 4

5. (a) If $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ represents one of the sets of three mutually perpendicular generators of the cone $5yz - 8zx - 3xy = 0$, find the equations of the other two. 4

(b) Find the equation to the cylinder with generators parallel to OZ and which passes through the curve of intersection of the surfaces represented by $x^2 + y^2 + 2z^2 = 12$ and $x + y + z = 1$. 4

UNIT–III

6. (a) Tangent planes are drawn to the conicoid $2x^2 + 3y^2 + 6z^2 = 1$ through the point (1, 1, 1). Prove that the perpendiculars to them from the origin generate the cone $(x + y + z)^2 = \frac{x^2}{2} + \frac{y^2}{3} + \frac{z^2}{6}$. 4

(b) Find the centre of the conic given by the equations

$2x^2 + 3y^2 + 5z^2 = 4,\ 2x + 6y + 15z - 59 = 0.$ 4

7. (a) Prove that the six normals from a point to a central conicoid lie on a come of second degree. 4

(b) Prove that the enveloping cylinder of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, whose generators are parallel to the lines $\frac{x}{0} = \frac{y}{\pm\sqrt{a^2 - b^2}} = \frac{z}{c}$ meet the plane $z = 0$ in circles. 4

UNIT–IV

8. (a) Show that the plane $8x - 6y - z = 5$ touches the paraboloid $\frac{x^2}{2} - \frac{y^2}{3} = z$ and find the point of contact. 4

(b) Show that the two confocal paraboloids cut everywhere at right angles. 4

9. (a) Find the equations of the generating lines of the hyperboloid $yz + 2zx + 3xy + 6 = 0$ which pass through the point $(-1, 0, 3)$. 4

(b) Reduce the equation $2x^2 - 7y^2 + 2z^2 - 10yz - 8zx - 10xy + 6x + 12y - 6z + 5 = 0$ the standard form and show that it represent a cone. 4