Roll No. .... Total Pages : 4

GSM/D-20 **891**

ADVANCED CALCULUS

Paper - BM-231

Time allowed : 3 Hours *Maximum Marks : 27*

Note : Attempt **five** questions in all, selecting at least one question from each unit. Question No. **1** is compulsory. All questions carry equal marks.

Compulsory Question

1. (i) State the Darboux theorem. 1

(ii) State Schwarz's theorem. 1

(iii) Define principal normal and binormal. ½

(iv) Define screw-curvature. What is its magnitude. ½

UNIT-I

2. (i) Show that every uniformly continuous function on closed and bounded interval is continuous. Is converse true ? Justify your answer. 3

(ii) Examine the applicability of Rolle's theorem for the function $f(x) = (x - 1)^{2/5}$ on [0, 3]. 3

3. (i) Show that the function

$$f(x) = \begin{cases} x \cos \frac{1}{x} & ; \quad x \neq 0 \\ 0 & ; \quad x = 0 \end{cases}$$

is continuous for every real x. 3

(ii) Find the values of a and b so that

$$\lim_{x \to 0} \frac{x(1 + a \cos x) - b \sin x}{x^3}$$

may be equal to 1. 3

UNIT-II

4. (i) Show that $f(x, y) = \sqrt{|xy|}$ is continuous at [0, 0]. 3

(ii) State and prove Euler's theorem on homogenous function. 3

5. (i) Let $f : \mathrm{R}^2 \to \mathrm{R}$ be defined as :

$$f(x, y) = \begin{cases} \dfrac{xy}{x^2 + y^2} & ; \quad (x, y) \neq (0, 0) \\ 0 & ; \quad (x, y) = (0, 0) \end{cases}$$

Show that $\lim\limits_{(x, y) \to (0, 0)} f(x, y)$ does not exist. 3

(ii) If A, B, C are the angles of a triangle such that $\sin^2 A + \sin^2 B + \sin^2 C = \text{constant}$.

Prove that $\dfrac{\partial A}{\partial B} = \dfrac{\tan C - \tan B}{\tan A - \tan C}$. 3

UNIT-III

6. (i) Show by an example that second order partial derivative of a function may exist at point but the function is not continuous thereat. 3

   (ii) In a plane triangle ABC, find the maximum value of cosA cosB cosC. 3

7. (i) Show that the function :

$$f(x, y) = \begin{cases} \dfrac{x^2y^2}{x^4 + y^4} & ; \quad (x, y) \neq (0, 0) \\ 0 & ; \quad (x, y) = (0, 0) \end{cases}$$

   is not differentiable at the origin. 3

   (ii) Find the maximum and minimum distance of the point (3, 4, 12) from the sphere $x^2 + y^2 + z^2 = 1$. 3

UNIT-IV

8. (i) Show that the curve

$$\vec{r} = \left(t, \frac{1+t}{t}, \frac{1-t^2}{t} \right)$$

   lies on a plane. 3

   (ii) Find the radius of curvature for the curve: 3

$$\vec{r} = 3a \cos 2\theta\, \hat{i} + 4c \sin^3\theta\, \hat{j} + 4c \cos^3\theta\, \hat{k} \text{ at } \theta = \frac{\pi}{4}.$$

9. (i) Show that the radius of spherical curvature of a circular helix

$x = \text{a} \cos\theta$, $y = \text{a} \sin\theta$, $z = \text{a}\, \theta \cot\alpha$

is equal to the radius of circular curvature. 3

(ii) Find the evolutes of circular helix $x = \text{a} \cos u$, $y = \text{a} \sin u$, $z = au \tan\alpha$. 3