Roll No. .... Total Pages : 04

GSE/D-21 746

CALCULUS
BM-112

Time : Three Hours] [Maximum Marks : 26

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. **1** is compulsory.

Compulsory Question

1. (i) Prove that $|\sin x|$ is continuous. **1**

(ii) State Taylor's theorem with Lagrange's form of remainder after n terms. **1**

(iii) Define radius of curvature. **1**

(iv) Evaluate $\int_0^{\pi/2} \sin^6 \theta \, d\theta$. **2**

(v) What is the axis of revolution ? **1**

Section I

2. (a) Expand $\tan x$ by Maclaurin's theorem as x^5 and hence find the value of tan 46°30' upto four decimal places. **2½**

(b) If $y=\left(\sin^{-1}x\right)^2$, prove that : **2½**

$$\left(1-x^2\right)y_{n+2}-(2n+1)xy_{n+1}-n^2y_n=0$$

3. (a) State and prove Maclaurin's theorem with Cauchy's form of remainder. **2½**

(b) Prove that : **2½**

$$\log\left(x+\sqrt{1+x^2}\right)=x-\frac{1}{2}.\frac{x^3}{3}+\frac{1.3}{2.4}.\frac{x^5}{5}-......$$

Section II

4. (a) Show that the four asymptotes of the curve $xy\left(x^2-y^2\right)+25y^2+9x^2-144=0$ cut it again in eight points on an ellipse whose eccentricity is $\frac{4}{5}$. **2½**

(b) Find the asymptote of the curve : **2½**

$$r\cos 2\theta=a\sin 3\theta$$

5. (a) Find P, the radius of curvature for the curve : **2½**

$$x=a\cos^3\theta,\ y=\sin^3\theta$$

(b) If C_x and C_y be the chord of curvature parallel to co-ordinate axes at any point of the curve $y = ae^{x/a}$, prove that : **2½**

$$\frac{1}{C_{x^2}} + \frac{1}{C_{y^2}} = \frac{1}{2aC_x}$$

Section III

6. (a) Trace the curve : **2½**

$$x = a(\theta + \sin\theta),\ y = a(1 + \cos\theta)$$

(b) If $u_n = \int_0^{\pi/4} \tan^n x\, dx$, show that : **2½**

$$u_n + u_{n-2} = \frac{1}{n-1}$$

Hence evaluate u_5.

7. (a) Find the length of the arc $x^2 + y^2 - 2ax = 0$ in the first quadratant. **2½**

(b) Find the intrinsic equation of the cycloid $x = a(t + \sin t),\ y = a(1 - \cos t)$ and prove that $s^2 + p^2 = 16a^2$. **2½**

Section IV

8. (a) Find the area common to the circle $x^2 + y^2 = 4$ and the ellipse $x^2 + 4y^2 = 9$. **2½**

(b) Show that the area of the region included between the cardioids $r = a(1 + \cos\theta)$ and $r = a(1 - \cos\theta)$ is $\frac{a^2}{2}(3\pi - 8)$. **2½**

9. (a) Find the volume of the solid of revolution obtained by rotating the area included between the curve $y^2 = x^3$ and $x^2 = y^3$ about the x-axis. **2½**

(b) Find the centroid of the semi-circular region of radius r by Pappus theorem. **2½**