Roll No. .... **Total Pages : 04**

GSE/D-21 747

SOLID GEOMETRY

BM-113

Time : Three Hours] [Maximum Marks : 27

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory.

Compulsory Question

1. (a) Find the centre of the conic : **1**

$$13x^2 - 18xy + 37y^2 + 2x + 14y - 2 = 0$$

(b) Find the equation of the sphere on the join of (–1, 3, 2) and (5, 7, –6) as diameter. **1**

(c) Show that the line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$, where $l^2 + 2m^2 - 3n^2 = 0$ is a generator of the cone $x^2 + 2y^2 - 3z^2 = 0$. **1**

(d) Find the equation of the plane which cuts the conicoid $x^2 + 4y^2 - 5z^2 = 1$ in a conic with centre (2, 3, 4). **1**

(e) Define confocal conicoids. **1**

Unit I

2. Find the lengths of the axes, the centre, the eccentricity and the equation of the axes of the conic $5x^2 - 24xy - 5y^2 + 14x + 8y - 16 = 0$. Also find foci and directrices. **5½**

3. (a) Find the conics confocal with $x^2 + 2y^2 = 2$, which passes through the point (1, 1). **3**

(b) Find the equation of the parabola which touches the conic : **2½**

$$x^2 + xy + y^2 - 2x - 2y + 1 = 0$$

at the point where it is cut by the line $x + y + 1 = 0$.

Unit II

4. (a) Find the equation of the sphere which passes through the circle $x^2 + y^2 + z^2 = 5$, $x + 2y + 3z = 3$ and touches the plane $4x + 3y - 15 = 0$. **2½**

(b) Find the equation of the right circular cone whose vertex is at the origin, axis the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$, and which has a vertical angle of 60°. **3**

5. (a) A plane ABC, whose equation is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ meets the axes in A, B, C. Find the equations to determine the circumcircle of the triangle ABC and obtain the co-ordinates of its centre. **2½**

(b) Find the equation of the enveloping cylinder of the sphere $x^2+y^2+z^2-2x+4y=1$ having its generators parallel to the line $x = y = z$. **3**

Unit III

6. (a) Prove that the sum of the squares of the reciprocals of any three mutually perpendicular diameters of an ellipsoid is constant. **2½**

(b) The section of the enveloping cone of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$, having vertex at P, by the plane $z = 0$ is a rectangular hyperbola. Find the locus of P. **3**

7. (a) Find the condition that the plane $lx+my+nz=1$ should touch the ellipsoid $\frac{x^2}{\alpha^2}+\frac{y^2}{\beta^2}+\frac{z^2}{\gamma^2}=1$. **2½**

(b) Find the equations of the polar of the line $\frac{x-1}{5}=\frac{y-3}{7}=\frac{z+5}{2}$ w.r.t. the conicoid $x^2+3y^2-7z^2=21$ in symmetrical form. **3**

Unit IV

8. Reduce the equation : **5½**

$$11x^2+10y^2+6z^2-8yz+4zx-12xy+72x$$
$$-72y+36z+150=0$$

to the standard form and show that it represents an ellipsoid and find the equations of the axes.

9. (a) Prove that there are five points on an elliptic paraboloid, the normals at which pass through a given point (α, β, γ). **2½**

(b) Find the equations to the generators of the hyperboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$, which pass through the point $(a\cos\alpha, b\sin\alpha, 0)$. **3**