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GSQ/D-21 **1062**

REAL ANALYSIS

Paper–BM-351

Time : Three Hours] [Maximum Marks : 40

Note : Attempt *five* questions in all, selecting *one* question from each Section. Q. No. 1 is compulsory.

Compulsory Question

1. (a) State First Mean Value theorem. 2

(b) Examine the convergence of $\int_0^1 \frac{dx}{x}$. 2

(c) Define open and closed sphere in a metric space. 2

(d) State Baire Category theorem. 2

SECTION–I

2. (a) Every bounded function is Riemann-Integrable. Prove or disprove. 4

(b) Show that $f(x) = x$ is integrable on $[a, b]$ and

$$\int_a^b f(x)\, dx = \frac{b^2 - a^2}{2}.$$ 4

3. (a) Using definition, evaluate $\int_0^{\pi/2} \sin x dx$. 4

(b) State and prove fundamental theorem on integral calculus. 4

SECTION–II

4. (a) Show that the integral $\int_0^{\frac{\pi}{2}} \frac{\sin mx}{x^n} dx$ exists iff $n < m + 1$. 4

(b) Evaluate $\int_0^a \frac{\log(1+ax)}{1+x^2} dx,\ a > 0$. 4

5. (a) Examine the convergence of $\int_{-a}^{a} \frac{x\,dx}{\sqrt{a^2 - x^2}}$. 4

(b) Using Frullani's theorem, show that

$$\int_0^\infty \frac{\tan^{-1} ax - \tan^{-1} bx}{x} dx = \frac{\pi}{2} \log\left|\frac{a}{b}\right|.$$ 4

SECTION–III

6. (a) Prove that in any metric space, every closed sphere is a closed set. 4

(b) Give an example of a space which is semi-metric space but not a metric space. 4

7. State and prove Cantor's Intersection theorem. 8

SECTION–IV

8. (a) Prove that the function $f : \mathrm{R} \to \mathrm{R}$ defined by $f(x) = x^2$ is not uniformly continuous on R. 4

(b) Prove that $f : (\mathrm{X}, d) \to (\mathrm{Y}, d^*)$ is continuous iff $\overline{f^{-1}(\mathrm{B})} \subseteq f^{-1}(\overline{\mathrm{B}})$. 4

9. (a) Prove that a compact subset of a metric space is closed and bounded. 4

(b) Prove that every compact metric space is complete. 4