Roll No. ..... **Total Pages : 04**

GSE/D-21 783

CALCULUS

BM-112

Time : Three Hours] [Maximum Marks : 40

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory.

Compulsory Question

1. (a) Evaluate : **2**

$$\lim_{n\to 0} \frac{x}{|x|}$$

(b) Evaluate : **2**

$$\lim_{n\to\infty} \frac{\Sigma n^2}{n^3}$$

(c) Find the radius of curvature at the origin for the curve : **2**

$$x^3 + y^2 - 2x^2 + 6y = 0$$

(d) What is a singular point ? **1**

(e) Define Quadrature. **1**

Unit I

2. (a) State and prove Taylor's theorem with Cauchy's form of remainder after n terms. **4**

(b) Prove that the function 'f' defined as :

$$f(x, y) = \begin{cases} x \sin \frac{1}{x}; & \text{if } x \neq 0 \\ 0; & \text{if } x = 0 \end{cases}$$

is continuous at $x = 0$ but not derivable at 0. **4**

3. (a) If $y = \frac{x^2}{(x-1)^3 (x-2)}$, find y_n. **4**

(b) If $y = \left[x + \sqrt{1+x^2} \right]^m$, find $y_n(0)$.

Unit II

4. (a) Find the asymptotes of $r \cos\theta = a \cos 2\theta$. **4**

(b) Find all the asymptotes of the curve : **4**

$$(x-y)^2 (x-2y)(x-3y) - 2a\left(x^3 - y^3\right) - 2a^2 (x-2y)(x+y) = 0$$

5. (a) If P_1 and P_2 are the radii of curvature at the extremities of a focal chord of a parabola whose latus rectum is ℓ, prove that :

$$(P_1)^{-2/3} + (P_2)^{-2/3} = (\ell)^{-2/3}$$

(b) Show that the radius of curvature for the cardioid $r = 1(1 - \cos\theta)$ is $\frac{2}{3}\sqrt{2ar}$ and prove that $\frac{P^2}{r}$ is constant. **4**

Unit III

6. (a) Trace the curve $r = a(1 + \cos\theta)$. **4**

(b) Evaluate $\int_0^\infty x^n e^{-x} dx$, where n is a positive integer. **4**

7. (a) If $u_n = \int \cos n\theta \operatorname{cosec}\theta \, d\theta$, prove that : **4**

$$u_n - u_{n-2} = \frac{2\cos(n-1)\theta}{(n-1)}$$

(b) Show that the length of the curve : **4**

$x^2(a^2 - x^2) = 8a^2y^2$ is $\pi a\sqrt{2}$

Unit IV

8. (a) Find the area common to the parabola $y^2 = 4x$ and $x^2 = 4ay$. **4**

(b) Show that the area of the region included between the cardioids $r = a(1 + \cos\theta)$ and $r = a(1 - \cos\theta)$ is $\frac{a^2}{2}(3\pi - 8)$. **4**

9. (a) Show that the surface area of the solid of revolution of $r = a(1+\cos\theta)$ about the initial line is $\frac{32}{5}\pi a^2$.

4

(b) Find the centroid of the quadrant of a circular arc.

4