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GSM/D-21 888

PARTIAL DIFFERENTIAL EQUATIONS

Paper–BM-232

Time Allowed : 3 Hours] [Maximum Marks : 26

Note : Attempt **five** questions in all, selecting **one** question from each Unit. Question No. **1** is compulsory.

Compulsory Question

1. (i) Find the partial differential equation by eliminating the arbitrary constants from the relation : 1

$$z = ax + by + a^2 + b^2$$

(ii) Find the complete integral of $(p + q)(z - px - qy) = 1$. 1

(iii) Solve $(D^2 + 2DD' + D'^2)z = e^{2x+3y}$. 2

(iv) Classify the partial differential equation : 1

$$\frac{\partial^2 z}{\partial x^2} + 6\frac{\partial^2 z}{\partial x \partial y} + 9\frac{\partial^2 z}{\partial y^2} = 0$$

(v) Write one dimensional and two dimensional wave equation. 1

UNIT-I

2. (a) From the partial differential equation by eliminating arbitrary functions from $z = f(xy/z)$. 2½

(b) Solve $p \tan x + q \tan y = \tan z$. 2½

3. (a) Examine whether the system of partial differential equations are compatible or not ? If compatible, find their common solution : 2½

$$\frac{\partial z}{\partial x} = 7x + 18y - 1$$

$$\frac{\partial z}{\partial x} = 9x + 11y - 2$$

(b) Find the complete integral of $z^2 = 1 + p^2 + q^2$ by using Charpit's method. 2½

UNIT-II

4. (a) Solve $(D^3 - 4D^2D' + 4D'^2D)Z = 4\sin(2x + y)$. 2½

(b) Solve $(D^2 - D'^2 + D + 3D' - 2)Z = e^{x-y} - x^2y$. 2½

5. (a) Solve $(D^2 - 2DD' + D'^2)Z = 12xy$. 2½

(b) Solve : 2½

$$x^2\frac{\partial^2 z}{\partial x^2} - 4xy\frac{\partial^2 z}{\partial x\partial y} + 4y^2\frac{\partial^2 z}{\partial y^2} + 6y\frac{\partial z}{\partial y} = x^3y^4$$

UNIT-III

6. (a) Classify and reduce the equation

$\frac{\partial^2 z}{\partial x^2} - x^2\frac{\partial^2 z}{\partial y^2} = 0$ to canonical form. 2½

(b) Classify and reduce the equation $\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x\partial y} + 4\frac{\partial^2 z}{\partial y^2} = 0$

to canonical form and hence solve it. 2½

7. (a) Reduce the partial differential equation

$\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x\partial y} + 13\frac{\partial^2 z}{\partial y^2} - 9\frac{\partial z}{\partial y} = 0$ to canonical form. 2½

(b) Solve by using Monge's method $r + 5s + 6t = 0$. 2½

UNIT-IV

8. (a) Find the real characteristics of partial differential equation : 2½

$$y^2\frac{\partial^2 z}{\partial x^2} - x^2\frac{\partial^2 z}{\partial y^2} = 0$$

(b) Show that the solution of the Cauchy Problem for the equation : 2½

$\frac{\partial^2 z}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 z}{\partial t} = 0, c > 0$ satisfying $z(x, 0) = f(x)$ and $\left[\frac{\partial z}{\partial t}\right]_{t=0} = 0,$

is $z(x, t) = ½\{f(x - ct) + f(x + ct)\}$.

9. Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, which satisfies the following boundary condition

$u(o, y) = u(a, y) = u(x, o) = 0$ and $u(x, b) = \sin\left(\frac{n\pi x}{a}\right)$. 5